

# Costly external finance, reallocation, and aggregate productivity

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**Abstract** Empirical studies document that resource reallocation across production units plays an important role in accounting for aggregate productivity growth in the US manufacturing. Financial market frictions could distort the reallocation process and hence may hinder aggregate productivity growth. This paper studies the quantitative impact of costly external finance on aggregate productivity through resource reallocation across firms with idiosyncratic productivity shocks. A partial equilibrium model calibrated to the US manufacturing data shows that costly external finance causes inefficient output reallocation from high productivity firms to low productivity firms and as a result leads to a 1 percent loss in aggregate TFP.

**Key words** Costly external finance · Reallocation · Output-weighted aggregate productivity

## 1 Introduction

This paper studies the quantitative impact of financial frictions on aggregate productivity in a setting with heterogeneous firms. Recently there has been increasing interest in understanding the microeconomic dynamics of aggregate productivity growth. Corresponding to this literature is a surge of empirical work that exploits establishment-level data to explore the relationship between microeconomic productivity dynamics and aggregate productivity growth. Representative work includes Baily et al. (1992), Bartelsman and Dhrymes (1998), and Foster et al. (2001). A

common theme of these studies is to decompose aggregate productivity growth into several parts to characterize the contributions of within firm/plant productivity growth and reallocation, where the latter includes the contribution of reallocation among continuing production units and the impact of entry and exit.<sup>1</sup> Although results vary with the specific data sets and decomposition methodologies used, a uniform finding in these studies is an important role of reallocation in accounting for aggregate productivity growth in the US manufacturing. For instance, Foster et al. (2001) document that reallocation accounts for about half of overall multifactor productivity growth in the US manufacturing for the period 1977–1987.

Distortions in product, labor and credit market and policies can all slow down aggregate productivity growth by hindering the reallocation process among heterogeneous producers. However, works that explain and quantify these impacts remain few. Restuccia and Rogerson (2008) explore the quantitative impacts of policy distortions on aggregate productivity in a stationary equilibrium with heterogeneous plants. They show that idiosyncratic output taxes or subsidies cause misallocation of resources across heterogeneous plants and, as a result, lead to sizable decreases in output and measured TFP. This paper is along the same line of Restuccia and Rogerson (2008), while the distortions we focus on are financial frictions.

<sup>1</sup> Petrin and Levinsohn (2005) argue that the popular measurement of aggregate productivity growth adds a “reallocation” term to the traditional growth accounting measure and fails to use the correct weights in the aggregation so that they call into question the literature’s interpretation of “reallocation” as productivity growth. Instead, they propose a new method for separating real productivity growth within plants from reallocation effects and find that such reallocation effects are reasonably stable within industries and almost always positively impact aggregate productivity growth.

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Frictions in financial markets can also cause resource misallocation since they constrain a firm's ability to finance profitable investment opportunities. Consequently, financial frictions could impose a negative effect on aggregate productivity growth. This paper formulates a simple partial equilibrium model to quantitatively assess this adverse impact. We abstract from modeling the microfoundations of financial frictions. Instead, financial market imperfections are summarized into a simple external finance cost function capturing the basic idea that external funds are more costly than internal funds if financial imperfections are present (see Fazzari et al. 1988). The external finance cost function is incorporated into an otherwise standard dynamic optimization problem of an infinitely-lived firm that faces exogenous idiosyncratic productivity shocks in every period of life. Stationary equilibrium properties with a large number of such firms are then examined to explore the implications of costly external finance on reallocation and aggregate productivity.

Note that the baseline model does not consider firm entry and exit. That is, we focus on the impact of costly external finance on reallocation among incumbent firms. This is empirically relevant since reallocation among continuing establishments is itself an important contributor to productivity growth in US manufacturing, as many empirical studies have documented.<sup>2</sup> On the other hand, the abstraction from entry and exit is not a big deviation from the Compustat data we use to calibrate the model, as the Compustat firms are relatively large and mature and do not exhibit a lot of entry and exit.<sup>3</sup> Like many studies on external finance (see Whited 1992; Hennessy and Whited 2005 and Gomes 2001), the Compustat data are used to calibrate the model since they provide detailed firm-level financial information that is crucial for calibrating the model, such as debt, equity issuance, interest expenses, and so on. A richer data set for the US manufacturing like LRD lacks such information.<sup>4</sup> We do recognize the significant

role entry and exit may play as the other important component of reallocation, so a discussion is given in a later section to examine how the results of the baseline model may change if firm entry and exit is considered.

We choose key parameters of the model to match certain moments of the data that describe important dimensions of investment dynamics and external finance of firms. The parameterized model is simulated to obtain the stationary properties of the industry, which are then compared with the properties of a stationary equilibrium with costless external finance. The results show that costly external finance imposes a more severe negative impact on high-productivity firms than on low-productivity firms such that it leads to a reallocation of output shares from high productivity firms to low productivity firms. As a result, the output-weighted aggregate TFP is 1 percent less than it would be if external finance is costless. The magnitude of this loss in aggregate productivity is slightly higher than the mean annual growth rate of output-weighted aggregate TFP for the US manufacturing over the period of 1970s to early 1990s (see Foster et al. 2001).

A comparative static analysis shows that the loss in aggregate productivity due to costly external finance increases with the return to scale, persistence and volatility of idiosyncratic productivity shocks, as well as external finance costs. This suggests that our result may underestimate the quantitative impact of costly external finance on aggregate productivity growth since firms in the Compustat sample are likely to exhibit less variability in productivities than a richer data set like LRD would exhibit and face lower external finance costs than an average manufacturing firm. A re-calibration is desirable when a richer data set incorporating finance and performance information becomes available.<sup>5</sup> Another implication of the comparative statics is that the adverse impact of costly external finance could be worse in recessions due to a higher external finance premium and higher uncertainty during recessions.<sup>6</sup> In particular, we show in an experiment that the sharp rise in micro uncertainty (volatility of idiosyncratic productivity shocks) as well as external finance costs in the recent credit crunch could exacerbate the adverse impact of costly external finance dramatically.

We then discuss the robustness of the quantitative result to several variations. We show that adding firm entry and exit to the model and re-calibrating it to LRD (a crude

<sup>2</sup> Baily et al. (1992) find that reallocation of output shares to more productive plants within stayers accounts for nearly half of the TFP growth for the 1972–1977 period and about one third of the rapid productivity growth in the 1980s. Foster et al. (2001) find that reallocation within continuing plants accounts for 26 percent of overall multifactor productivity growth in US manufacturing for the 1977–1987 period.

<sup>3</sup> The Compustat data record the year a firm is deleted from the file and the reason for deletion. Among the reasons for deletion, bankruptcy and liquidation are regarded as closely related to firm exit from operation. During the period 1989–2003, which is the sample period of the data set we use to calibrate the model, firm deletion rate due to bankruptcy and liquidation is about 0.5 percent.

<sup>4</sup> A disadvantage of using Compustat data is it represents only about 1/3 of employment in the US, see Davis et al. (2006). Another disadvantage is the lack of young and small firms in Compustat data. As documented in empirical studies, young and small firms play an important role in reallocation.

<sup>5</sup> Currently, the Center for Economic Studies of the Bureau of the Census is linking the LRD to many other data sets, including public financial databases.

<sup>6</sup> Agency cost models, such as Bernanke et al. (1996), suggest that the external finance premium is countercyclical since it is inversely related to firms' net worth which is procyclical. Bloom et al. (2009) document a significant rise in both micro and macro uncertainty during recessions.

calibration) yield a larger loss in aggregate productivity due to costly external finance than suggested by the baseline model. Considering capital adjustment cost may alleviate or exacerbate the impact of costly external finance on aggregate productivity. The quantitative results from these variations, however, do not seem to deviate from the baseline result dramatically.

This study also has interesting implications for the impact of financial frictions on output growth. Much of this important literature adopts the framework of neoclassical growth models that abstract from heterogeneity in production units, and hence is more concerned with understanding the role of aggregate accumulation and how aggregate accumulation is affected by financial market frictions. The empirical studies mentioned earlier, however, suggest that it is not only the level of factor accumulation that matters for aggregate output but also how these factors are allocated across heterogeneous production units. In our model, costly external finance decreases aggregate output through two channels. One is the traditional channel-capital accumulation. Costs associated with external finance makes investment more expensive and hence decrease aggregate capital accumulation. The other channel is through resource reallocation across heterogeneous firms which results in a lower aggregate productivity. Our results show that with costly external finance, the reallocation leads to 0.3 percent loss in aggregate output, which is about a third of its impact on aggregate productivity. The small magnitude of this effect may suggest that the impact of costly external finance on aggregate output through reallocation is not quite significant, though a thorough evaluation of the relative importance of aggregate accumulation and reallocation calls for a general equilibrium analysis and a richer data set for calibration purpose.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 details the calibration and simulation methods. Section 4 describes the results from the baseline model. Section 5 discusses the robustness of the main quantitative result to several variations. And Section 6 gives some final remarks.

## 2 The model

The analysis is of partial equilibrium type, in that it focuses on the dynamic optimization problem of a single firm which takes prices as given. When assessing the aggregate implications of costly external finance, a large number of such firms are considered. In view of the main question to be addressed, we assume constant aggregate prices and abstract from aggregate shocks to focus on a stationary

analysis. A brief discussion is given in Sect. 5 on the robustness and limitations of the analysis.

The firm is infinitely lived. That is, we exclude firm entry and exit from the analysis. In a later section, we discuss how the results would change if considering firm entry and exit. In period  $t$ , the firm's operating cash flow is generated by a profit function given by

$$\pi(k_t, z_t) = e^{z_t} k_t^\alpha, \quad \alpha < 1. \quad (1)$$

This profit function can be viewed as a reduced form that has optimized out inputs other than capital, as in Cooper and Haltiwanger (2006). Here,  $k_t$  is the firm's capital stock at the beginning of period  $t$ . Capital depreciates at rate  $\delta$  and must be decided one period in advance. The profit function exhibits decreasing return to scale, with  $\alpha < 1$  measuring its curvature.<sup>7</sup>  $e^{z_t}$  is the firm's idiosyncratic total factor productivity (TFP) in period  $t$ , where  $z_t$  is assumed to follow an AR(1) process given by

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}$$

with the productivity shock  $\varepsilon$  following a truncated normal distribution with zero mean, standard deviation of  $\sigma$  and finite support  $[-10\sigma, 10\sigma]$ .

Given the reduced form profit function in (1), it should be pointed out that more accurately  $z_t$  indicates the firm's profitability. As emphasized in Foster, Haltiwanger and Syverson (2008), a firm's profitability may also be associated with several factors other than productivity, such as idiosyncratic demand or cost shocks. It is challenging, however, to separate physical productivity shocks from profitability shocks, due to the lack of data on micro-level prices.<sup>8</sup> Here, we take the position that all of the profitability shocks are due to productivity shocks. In view of this simplification, in the calibration (Sect. 3), we identify the productivity shocks by moments that describe investment dynamics of firms rather than moments capturing profitability. The rationale is that firms' investment may be more responsive to variations in technological efficiency than to transitory demand shocks.

The firm can finance its investment in capital by internal funds or borrowing from the financial market.

<sup>7</sup> This does not necessarily imply decreasing return to scale in the underlying production function. Alternatively, as in Cooper and Ejarque (2003) and Bloom (2009), if a firm has constant return to scale production function and faces iso-elastic demand curve, its profit function would exhibit decreasing return to scale. So  $\alpha$  is also referred as the revenue return to scale; a lower  $\alpha$  indicates higher mark-up or market power.

<sup>8</sup> Most empirical research on productivity using business microdata, including the studies we cited in the Introduction, had to measure output using revenue data so that their productivity measures embody price differences which may reflect idiosyncratic demand shifts or variations in market power. Foster et al. (2008), using a new data set with price observations, is one of the few studies that are able to compute physical productivity directly.

As in Gomes (2001), we assume that financial market imperfections exist and are summarized with a simple external finance cost function that takes the linear form given by

$$\lambda = \lambda_0 + \lambda_1 \times \text{amount of external funds.}$$

That is, there is a fixed cost  $\lambda_0$  and per unit cost  $\lambda_1$  associated with external finance. This specification is intended to capture a variety of costs of going to financial market to raise capital, which may include the fixed and variable costs of public stock offerings, costs of monitoring the firm and the discounted present value of any premia associated with external debt and equity finance. Clearly the firm will choose to use external finance only when it exhausts internal funds and current investment opportunities justify the additional cost of external funds.

The firm's problem is to choose its capital stock to maximize its expected discounted sum of future net cash flow, taking as given a constant price of capital input  $p$ . The problem has the following recursive formulation.

$$V(k, z) = \max_{k' \geq 0} \{ \pi(k, z) - pi(k, k') - \lambda_0 I\{pi(k, k') > \pi(k, z)\} - \lambda_1 \max\{pi(k, k') - \pi(k, z), 0\} + \beta E_{z|z} V(k', z') \} \quad (2)$$

where  $i(k, k') \equiv k' - (1 - \delta)k$  denotes the investment in capital, and  $I\{\cdot\}$  is an indicator function. The firm's present value, as expressed on the right-hand side of (2), is the sum of current net cash flow and expected discounted continuation value, where the former is current profits minus investment spending and financing costs.

Notice that in the model firms can only save through real assets (capital). We abstract from firm savings in cash holding or other financial assets. Allowing for these other forms of savings would give firms more means of transferring funds across periods, and consequently may alleviate firms' financing constraints due to costly external finance. However, this simplification should not be quantitatively significant for the question we aim to address, since in the data sample we use to calibrate the model capital expenditure accounts for 86 percent of total investing funds while funds used for cash holdings and short-term financial assets are only 7 percent. Also, in our calibration, we restrict investment to be capital expenditure alone.

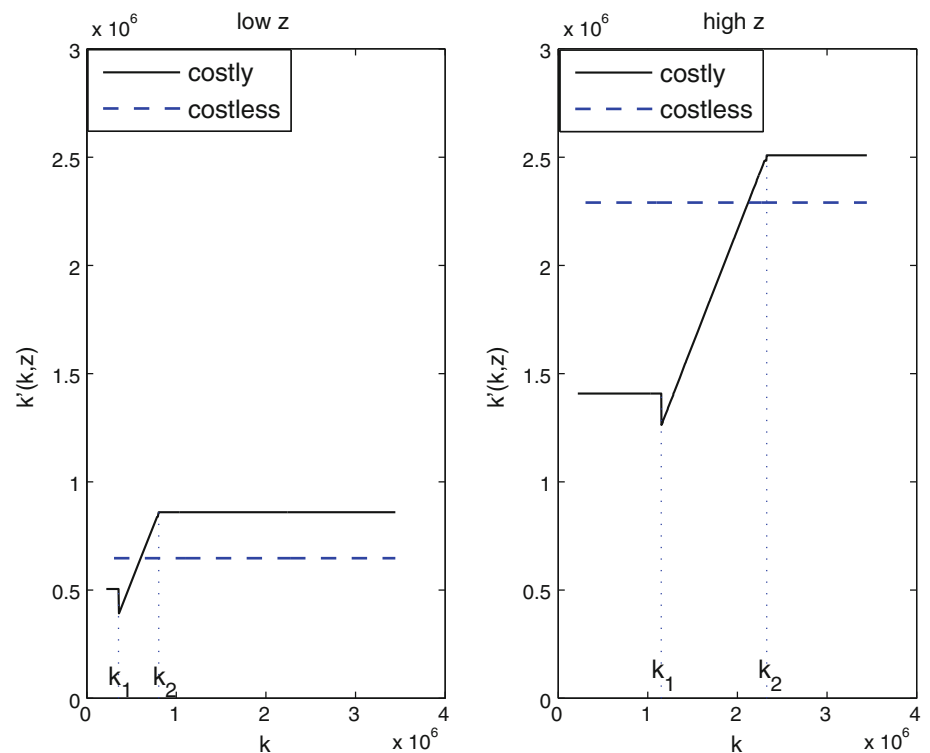
Applying standard arguments of dynamic programming, as established in Stokey and Lucas (1989), one can show that there exist a unique value function  $V(k, z)$  and a unique policy function  $k'(k, z)$  that solve (2). Figure 1 plots the policy function  $k'(k, z)$  for a low level of current productivity  $z$  and a high level of  $z$ . The figure is based on the baseline parameterization to be described in next section. In both plots, the solid line corresponds to the case of

costly external finance, and the dashed line corresponds to costless external finance. Figure 1 clearly shows that if external finance is costless ( $\lambda_0 = \lambda_1 = 0$ ),  $k'(k, z)$  is a function of current productivity  $z$  only, i.e., capital accumulation is independent of current capital stock. Costly external finance introduces dependence of  $k'$  on  $k$  in a pattern as follows. For a given current productivity level, if the firm's current capital stock is less than some critical level (denoted as  $k_1$  in Fig. 1), using external finance is profitable so that the firm resorts to external finance and optimal  $k'$  is independent of current  $k$ . If  $k$  is greater than this level while less than some other critical level (denoted as  $k_2$ ), since the profit function exhibits decreasing return to scale, current investment opportunities would not justify the additional cost of external finance and hence the firm's investment is constrained by its operating profit. On this region the optimal  $k'$  depends on current capital stock. If  $k$  is big enough (greater than  $k_2$ ), the firm's current profit is sufficient to finance desired level of investment so that its investment is no longer financially constrained and the unconstrained level of  $k'$  is independent of  $k$ . Figure 1 also shows that with costly external finance  $k'(k, z)$  may be discontinuous at  $k_1$ . This is due to the nonlinearity introduced by a fixed external finance cost.

Another finding from Fig. 1 is that the unconstrained level of  $k'$  with costly external finance is bigger than the optimal  $k'$  under costless external finance (these two are equal only if  $z$  is at the highest level). This result implies that with costly external finance firms have an incentive to over-accumulate capital when they are not financially constrained, a behavior similar to "precautionary saving" by households subject to borrowing constraints. That is, costly external finance leads to a distortion in firms' investment behavior. A comparison of the two plots shows that the constrained region with a high current productivity is larger than the constrained region with a low productivity (both  $k_1$  and  $k_2$  are larger with a higher  $z$ ), suggesting that high productivity firms are more seriously impacted by costly external finance. The intuition underlying this result is as follows. Productivity shocks are highly persistent under the baseline calibration, so firms with high productivity today are expected to have high productivity tomorrow and hence more likely to increase capital investment to take advantage of this productivity boom. Consequently they are more impacted by costly external finance. This property of the model will help explain why the presence of costly external finance has an adverse impact on aggregate productivity, as will be clear in a later section.

Figure 1 also implies that small firms (with smaller capital stock) resorts to external finance more often. This seems to contradict the commonly held belief that small firms are more financially constrained and rely on internal

**Fig. 1** Policy function for next period capital stock. In the computation, the productivity shock process is approximated with a 10-state Markov chain. Here, the low productivity refers to the third state, and the high productivity refers to the 9th state. Similar patterns hold for other choices of productivity levels. In both plots,  $k_1$  denotes the level of current capital stock below which the firm resorts to external finance while above which the firm finances investment by internal funds only, and  $k_2$  denotes the capital stock above which current profits are sufficient to finance the firm’s desired investment



**Table 1** External finance ratio by asset class, compustat manufacturing firms, 1989–2003

	External funds/sources of funds <sup>a</sup>	External funds/uses of funds
All firms	0.1077	0.1123
<\$ 250 million	0.9337	0.9660
\$250 million–\$ 1 billion	0.2593	0.2974
\$ 1–2 billion	0.1691	0.1844
>\$ 2 billion	0.0784	0.0800

<sup>a</sup> For definitions of external funds and sources and uses of funds, see Appendix A.1 of the working paper: <http://www.economics.unimelb.edu.au/research/wp/wp08/1044.pdf>

funds more heavily. We document external finance ratios by asset class for Compustat manufacturing firms during the 1989–2003 period and report them in Table 1. A strong negative relationship is found between external finance

<sup>9</sup> There is belief that high external finance ratios for small firms as shown in Table 1 are due to the fact that a lot of small firms in Compustat are young high-tech firms which are recently publicly listed and have very high equity financing. Since firm age information is not available in Compustat, we are not able to re-examine this relationship by controlling for firm age. But we re-calculate the external finance ratios by asset class for each of the 20 manufacturing industries and find that the negative relationship between external finance ratio and firm asset size holds for most industries and is particularly remarkable for some high-tech industries such as Chemicals & Allied Products (SIC code 2800), Industrial and Commercial Machinery and Computer Equipment (SIC code 3500), Electric and Electronic Equipment and Exchange Components (SIC

ratios and the total assets of firms. That is, smaller firms have higher external finance ratios than larger firms.<sup>9</sup> Since Compustat firms are mainly large mature firms, it’s not clear whether this relationship holds for all manufacturing firms. Nevertheless, this finding suggests that the commonly held belief may not hold uniformly in the data.

### 3 Calibration and simulation

To implement a quantitative analysis, we need to set values for parameters, including the relative price of capital good,  $p$ , the discount factor,  $\beta$ , the depreciation rate of capital,  $\delta$ , the return to scale,  $\alpha$ , the parameters describing the productivity shock,  $\rho$  and  $\sigma$ , and parameters in the external finance cost function,  $\lambda_0$  and  $\lambda_1$ . The data we use to estimate or calibrate the parameters is taken from the Compustat North American industry annual file. We only consider firms in the manufacturing sector (with SIC codes between 2000 and 3999) during the period of 1989 to 2003. This time period is chosen since there are substantial changes in the reporting and accounting methods since 1988. Observations with missing data are deleted from the sample. Similar to Whited (1992) and Gilchrist and

Footnote 9 continued  
code 3600), Measurement Instrument, Photo Goods and Watches (SIC code 3800). When we exclude these industries from the data sample, the negative relationship between firm size and external finance ratio still holds but is less remarkable than shown in Table 1.



**Table 2** Identification of parameters

Parameters	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\delta$	0.1	0.11	0.09	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$\alpha$	0.975	0.975	0.975	0.986	0.966	0.975	0.975	0.975	0.975	0.975	0.975
$\rho$	0.95	0.95	0.95	0.95	0.95	0.96	0.94	0.95	0.95	0.95	0.95
$\sigma$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.011	0.009	0.01	0.01
$\lambda_0$	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,100	900
Moments <sup>a</sup>											
<i>I/K</i>	0.1004	0.1106	0.0906	0.1020	0.1007	0.1008	0.1008	0.1006	0.1005	0.1002	0.1009
Avg. of <i>ik</i>	0.1118	0.1223	0.1010	0.1365	0.1079	0.1143	0.1106	0.1149	0.1094	0.1032	0.1117
Std. of <i>ik</i>	0.1396	0.1419	0.1342	0.2982	0.1113	0.1563	0.1280	0.1606	0.1248	0.0725	0.1391
Corr. of <i>ik</i>	0.1320	0.1351	0.1359	0.0917	0.1380	0.1304	0.1353	0.1361	0.1329	0.1275	0.1350
Extfin. <i>I</i>	0.0491	0.0492	0.0471	0.1401	0.0195	0.0692	0.0360	0.0743	0.0323	0.0155	0.0493

<sup>a</sup> *I/K*: aggregate investment rate, Avg. of *ik*: cross-sectional average of investment rates, Std. of *ik*: cross-sectional standard deviation of investment rates, Corr. of *ik*: auto-correlation of investment rates, Extfin./*I*: external finance ratio

Himmelberg (1995), we exclude observations with large changes in the book value of capital stock, considering that they may indicate expansions or contractions of firms at margins other than capital expenditure. Finally we end up with an unbalanced panel of firms from 1989 to 2003 with between 2,210 and 3,265 observations per year.<sup>10</sup>

First, we normalize  $p$  to 1. Following Cooper and Ejarque (2003), we set  $\beta$  to 0.95. The external finance cost function cannot be directly estimated, since data on total expenses of external finance are not available in Compustat. Using data on costs associated with new equity issuance, Smith (1977) and Altinkilic and Hansen (2000) estimate an external finance cost function of the same form as ours. Their estimates for  $\lambda_1$  are 0.028 and 0.0241, respectively. In our data sample, external finance mainly takes the form of debt finance rather than equity finance (equity finance is about 10 percent of total external finance), so we re-estimate this parameter by a panel regression of interest expenses of debt on debt issuance. It gives a similar result,  $\lambda_1 = 0.028$ . Since  $\lambda_0$  is sensitive to units of measure, it is estimated together with  $\alpha$ ,  $\delta$ ,  $\rho$  and  $\sigma$  to match five moments of the data.

The first moment is the mean annual investment rate defined as the ratio of total investment to total capital stock, which is 0.17 for the data sample. The second moment is the cross-sectional average investment rate, which is 0.22. The third moment is the cross-sectional standard deviation of investment rates, which is 0.19. The fourth moment is the autocorrelation of investment rates, which is 0.21. In constructing investment rates for each firm at each year, the book values of the gross capital stock are converted into its

replacement values following the perpetual inventory method described in Salinger and Summers (1983). The last moment is the fraction of total investment financed externally, i.e. the ratio of external finance used for investment to total investment. Compustat does not have enough information to directly calculate this moment. But it can be reasonably approximated by the ratio of total external finance to total uses of funds, which is 0.072, since in the data sample 86 percent of total uses of funds are for new capital purchase. These five moments are selected for their informativeness about the underlying structural parameters as well as their prominence in the literature.

To demonstrate that these five moments provide identification of the five parameters to be estimated, Table 2 presents how their values change with respect to small changes in each parameter. In the table, parameterization (1) is a benchmark parameterization, where the parameters are set to values commonly used in the literature. In particular, the annual depreciation rate  $\delta$  is set to 10%; the return to scale parameter  $\alpha$  is set to 0.975, a value close to the standard CRS assumption; parameters governing the productivity shocks  $\rho$  and  $\sigma$  are set to 0.95 and 0.01, respectively, values that are commonly used in the real business cycle literature; the fixed external finance cost  $\lambda_0$  is set to 1,000, a positive but very small number relative to the average amount of external finance in equilibrium. Parameterization (2) considers a 10% change in  $\delta$  relative to the benchmark with all other parameters unchanged, parameterization (3) and (4) consider a 1% change in  $\alpha$  and  $\rho$  respectively, and parameterization (5) and (6) consider a 10% change in  $\sigma$  and  $\lambda_0$ , respectively. The results indicate that the five moments we choose are sensitive to changes in the parameters. In particular, the investment rate (*I/K*) is very sensitive to changes in  $\delta$ , the cross sectional average and standard deviation of investment rates are sensitive

<sup>10</sup> For a detailed description of the data sample, see Appendix A.1 of the working paper: <http://www.economics.unimelb.edu.au/research/wp/wp08/1044.pdf>.

**Table 3** Baseline calibration

	Parameter	Value
Price of capital	$p$	1
Discount factor	$\beta$	0.95
Returns to scale	$\alpha$	0.8993
Depreciation rate	$\delta$	0.17
Persistence of shock	$\rho$	0.8767
Variability of shock	$\sigma$	0.0393
Fixed cost of external finance	$\lambda_0$	608.4139
Unit cost of external finance	$\lambda_1$	0.028
Matched moments	Data	Model
$I/K$	0.17	0.1703
Avg. of $ik$	0.22	0.1868
Std. of $ik$	0.19	0.1784
Corr. of $ik$	0.21	0.1632
Extfin. $I$	0.072	0.0724

to changes in all parameters, the autocorrelation of investment rates is sensitive to changes in  $\alpha$  and  $\rho$ , and the external finance ratio is very sensitive to changes in  $\alpha$ ,  $\rho$ ,  $\sigma$  and increases in  $\lambda_0$ . So we conclude that these moments provide identification of the parameters to be estimated.

Here is a brief description of the estimation procedure.<sup>11</sup> For arbitrary values of the parameters to be estimated, the productivity shock is approximated by a 10-state Markov chain following Tauchen (1986) and the firm’s problem is solved by value function iteration to obtain the policy function  $k'(k, z)$ . Using the policy function, an invariant distribution of firms over capital stock and productivity types,  $\mu(k, z)$ , is computed. Then we draw 20,000 firms from the invariant firm distribution and carry out the simulation for 15 periods (the data sample covers 15 years) to form an artificial panel data set. The five moments are computed for this artificial data set and compared with the corresponding data moments. This procedure is continued until the distance between the moments of the simulated data and the actual data moments is minimized. Considering the potential discontinuity introduced by the fixed external finance cost and the discretization of the state space, we use a simulated annealing algorithm as described in Goffe et al. (1994) to perform the minimization. Table 3 summarizes the estimated parameter values and matched moments.

The high degree of nonlinearities in the solution makes it hard to match all moments exactly. Nevertheless the approximation appears reasonably close, as shown in

<sup>11</sup> A more detailed description can be found in Appendix A.3 of the working paper: <http://www.economics.unimelb.edu.au/research/wp/wp08/1044.pdf>.

Table 3. The estimate of  $\alpha$  is 0.8993, suggesting that the revenue return to scale does not substantially depart from constant return to scale. This is consistent with the estimates of revenue returns to scale in recent studies such as Khan and Thomas (2003) and Bachmann et al. (2006). Cooper and Haltiwanger (2006) give a much lower estimate of revenue returns of about 0.6 using the LRD plant level data. The estimate of capital depreciation rate is 0.17, close to the value used in many studies that examine firm-level investment behavior using Compustat data: 0.15 in Gilchrist and Himmelberg (1995) and Cooper and Ejarque (2003), and 0.145 in Gomes (2001). Consistent with the literature,  $\rho$  is estimated to be close to 1, suggesting that productivity shocks are highly persistent. The estimate of the variability of productivity shocks  $\sigma$  is consistent with Gomes (2001) which also excludes capital adjustment cost, while much smaller than the estimate of Cooper and Haltiwanger (2006) that considers capital adjustment cost. Such difference reflects the fact that capital adjustment costs reduce the volatility of investment so that a higher volatility of productivity shocks would be needed to match the same cross-sectional variation in investment rates. The fixed cost of external finance  $\lambda_0$  is estimated to be about 608, which is about 0.2 percent of the average size of external finance in the stationary equilibrium. Cooper and Ejarque (2003) estimate a similar model with convex capital adjustment cost and fixed external finance cost alone, and obtain an estimate of  $\lambda_0$  close to zero.

### 4 Results

With the parameters determined, the question outlined in the Introduction can be addressed. This section summarizes the quantitative impacts of costly external finance on aggregate productivity, capital accumulation and output. Comparative statics are examined next and finally we briefly discuss the robustness of the quantitative results.

#### 4.1 Impact of costly external finance on aggregate productivity

In a setting with heterogeneous production units, aggregate productivity is typically defined as a weighted sum of establishment-level productivities, where the weights can be output shares or employment shares of establishments in total output or employment. Here we use output shares in aggregating firm-level TFPs to obtain aggregate productivity, which is referred as output-weighted aggregate productivity.

To evaluate the quantitative impact of costly external finance on aggregate productivity, we compute the output-weighted aggregate productivity and compare it with the

**Table 4** Quantitative impact of costly external finance

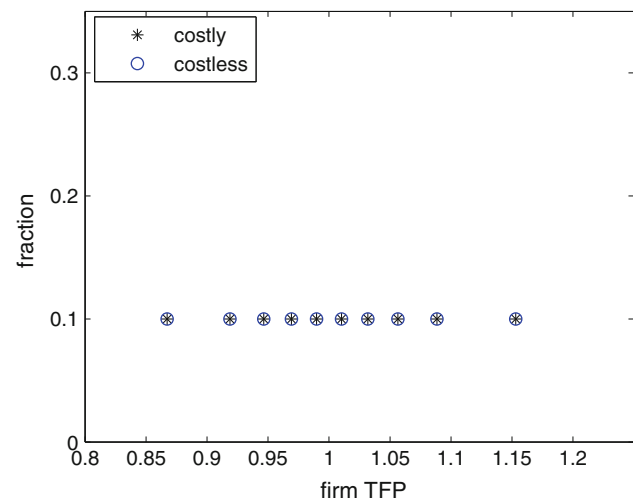
	Costly ext. finance	Costless ext. finance	Ratio (costly/costless)
Average productivity	1	1	1
Output-weighted productivity	1.0395	1.0496	0.9904
Aggregate capital stock <sup>a</sup>	$1.2290 \times 10^6$	$1.3255 \times 10^6$	0.9272
Aggregate output	$3.0562 \times 10^5$	$3.2814 \times 10^5$	0.9314

<sup>a</sup> The aggregates are based on a unit measure of firms in both cases

productivity measure that would be obtained if external finance is costless, i.e., if all parameter values are the same as in Table 3 except  $\lambda_0 = 0$  and  $\lambda_1 = 0$ . To compute the output-weighted aggregate productivity, a distribution of output shares across different productivity types is needed. The invariant measure of firms over capital stock and productivity,  $\mu(k, z)$ , enables us to do so. As reported in Table 4, the output-weighted aggregate productivity with costly external finance is 1.0395, while its costless counterpart is 1.0496. These results imply a 1 percent loss in aggregate productivity due to costly external finance. Despite the small magnitude, this adverse impact is not quantitatively insignificant. According to Foster et al. (2001), the output-weighted multifactor productivity growth over a five-year period in US manufacturing is 2.7, 7.32, and 3.3 percent for 1977–1982, 1982–1987 and 1987–1992 respectively, implying an annual growth rate much less than 1 percent for most years during the 15-year period. Next, we examine this result from several aspects by comparing the two steady state distributions with costly and costless external finance.

First, as illustrated in Fig. 2, the productivity distributions with costly or costless external finance are the same: firms with each of the 10 productivity types account for 10 percent of all firms. So the average productivity is 1 in both cases, as reported in Table 4, implying that the change in aggregate productivity due to within firm productivity growth is zero. Therefore, the 1 percent loss in output-weighted aggregate productivity due to costly external finance is completely through the reallocation of output shares across firms.<sup>12</sup> This is shown clearly in Fig. 3, which plots the distribution of output shares across productivity types for the two cases. Note that with costly external finance, the output shares of firms with high level productivities are smaller than their costless counterparts, while the output shares of firms with low productivities are larger than their costless counterparts. It follows that the

<sup>12</sup> According to a widely used decomposition methodology of aggregate productivity growth, the change in aggregate productivity can be decomposed into several parts characterizing the relative contributions of within establishments productivity growth, reallocation, and net entry. See Baily et al. (1992) for an application of this decomposition method, and Foster et al. (2001) for a discussion of various problems it's subject to and other decomposition methods.

**Fig. 2** Firm distribution over productivity types

presence of costly external finance leads to a shift of output shares from high productivity firms to low productivity firms and hence results in a lower aggregate productivity. The driving force underlying this result is the distortion in firms' investment behavior due to costly external finance. As discussed earlier, the adverse effect that costly external finance hinders capital accumulation is more severe for high productivity firms than for low productivity firms. Consequently, the output of high productivity firms is more seriously impacted by costly external finance than low productivity firms and as a result costly external finance leads to a reallocation of output shares from high productivity to low productivity firms.

Finally, Fig. 4 plots the firm distribution over capital stock in the two cases. If external finance is costless, firms with the same productivity will have the same capital stock, and as a result the firm distribution is a uniform distribution over the 10 efficient levels of capital stock corresponding to the 10 productivity types. While with costly external finance, since firms are financially constrained in achieving their efficient size, the resulting firm distribution is skewed to the right, with a majority of firms having low capital stock while only a small fraction of firms having very high capital stock. This feature of the model is consistent with the data.



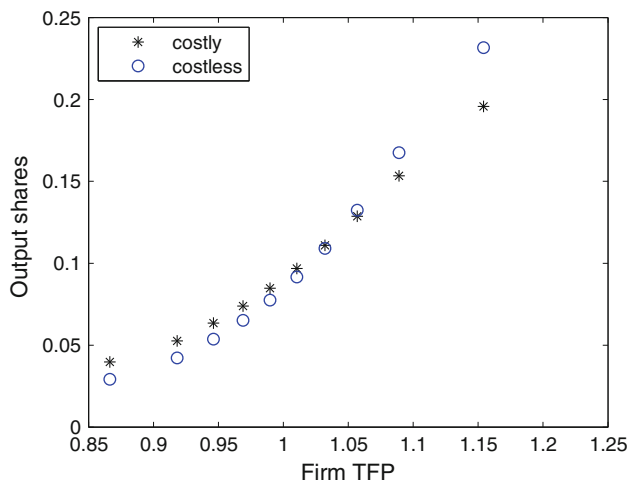


Fig. 3 Distribution of output shares over productivity types

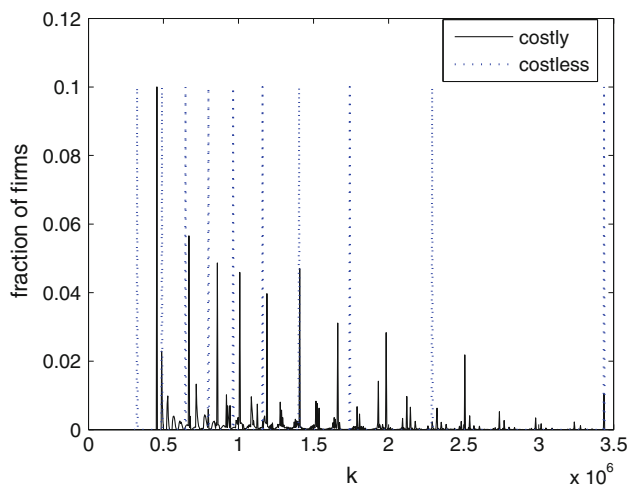


Fig. 4 Firm distribution over capital stock

#### 4.2 Impact of costly external finance on output through reallocation

According to Table 4, costly external finance decreases aggregate output by 6.9 percent. This is achieved through two channels. One is the traditional channel—capital accumulation. As shown in Table 4, costly external finance decreases aggregate capital accumulation by 7.3 percent. These results, however, hinge on a partial equilibrium analysis. That is, we keep the price of capital unchanged,  $p = 1$ , when solving the costless problem. In a general equilibrium setting, aggregate capital accumulation and aggregate output may be lower than reported in Table 4 since the price of capital goods may increase to discourage investment as investment demand rises, and hence the adverse impact of costly external finance on aggregate output may be smaller than suggested in Table 4.

Here, our focus is on the impact of costly external finance on output through the second channel—a lower

aggregate productivity resulting from resource misallocation. To quantify this impact, we do the following experiment. When solving the costless problem, we vary the price of capital good  $p$  such that aggregate capital stock is the same as in the equilibrium with costly external finance. In this way, we keep the aggregate capital accumulation the same in both cases. Any change in aggregate output is fully due to changes in aggregate productivity. The result is summarized in Table 5. First note that the output-weighted aggregate productivity with costless external finance is not affected by the change in capital price so that the loss in aggregate productivity due to costly external finance is the same as in Table 4.<sup>13</sup> Table 5 shows that the 1 percent decrease in aggregate productivity due to costly external finance leads to about 0.3 percent decrease in aggregate output, which seems a small negative impact on aggregate output.

A large literature that attempts to explore the relationship between financial market frictions and output growth adopts the framework of neoclassical growth models that abstracts from heterogeneity in production units and has been concerned with the role of aggregate capital accumulation and how it is affected by financial frictions. The role of reallocation is completely neglected. With heterogeneous firms, this model can characterize both roles of aggregate accumulation and reallocation, where the latter is characterized by the change in output-weighted aggregate productivity. The quantitative result seems to suggest that the impact of costly external finance on aggregate output through reallocation is not quite significant as far as the Compustat data is concerned. A thorough evaluation of the quantitative impact of costly external finance on output growth, however, calls for a general equilibrium analysis and a richer data set for calibration purpose.

#### 4.3 Comparative statics

In this section we examine how the impacts of costly external finance vary with key primitives of the model: the price of capital, the return to scale, the persistence and volatility of productivity shocks, and external finance costs. With a variation in a particular parameter while other parameters at their baseline values, the firm’s problem is re-solved and the model is simulated to obtain the four moments: cross-sectional mean, standard deviation and autocorrelation of investment rates, and fraction of total investment financed externally (Aggregate investment rate, mainly determined by the depreciation rate of capital, is about 0.17 in all these scenarios). The corresponding

<sup>13</sup> In fact, we proved in the working paper that the output-weighted aggregate productivity with costless external finance is independent of the price of capital.

**Table 5** Quantitative impact of costly external finance through reallocation

	Costly ext. finance	Costless ext. finance	Ratio (costly/costless)
Price of capital good	1	1.0076	0.9925
Average productivity	1	1	1
Output-weighted productivity	1.0395	1.0496	0.9904
Aggregate output	$3.0562 \times 10^5$	$3.0657 \times 10^5$	0.9969

**Table 6** Comparative statics

	Avg. of $i/k$	Std. of $i/k$	Corr. of $i/k$	Extfin./ $I$	K	Y	TFP <sup>a</sup>
$p = 0.8$	0.1878	0.1786	0.1751	0.2476	0.9274	0.9317	0.9907
$p = 1$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$p = 1.2$	0.1856	0.1777	0.1179	0.0521	0.9264	0.9302	0.9893
$\alpha = 0.85$	0.1787	0.1214	0.1177	0	0.9678	0.9706	0.9938
$\alpha = 0.8993$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$\alpha = 0.95$	0.2377	0.4317	0.1217	0.2849	0.7360	0.7429	0.9838
$\sigma = 0.03$	0.1796	0.1294	0.1469	0.0217	0.9559	0.9582	0.9937
$\sigma = 0.0393$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$\sigma = 0.05$	0.2015	0.2597	0.1400	0.1623	0.8972	0.9034	0.9878
$\rho = 0.84$	0.1833	0.1553	0.1482	0.0480	0.9361	9395	0.9911
$\rho = 0.8767$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$\rho = 0.9$	0.1906	0.2011	0.1581	0.0982	0.9193	0.9243	0.9901
$\lambda_0 = 0$	0.1875	0.1790	0.1757	0.0809	0.9273	0.9315	0.9907
$\lambda_0 = 608.4$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$\lambda_0 = 1000$	0.1868	0.1795	0.1528	0.0717	0.9264	0.9305	0.9903
$\lambda_1 = 0.02$	0.1906	0.2052	0.1378	0.1126	0.9346	0.9388	0.9923
$\lambda_1 = 0.028$	0.1868	0.1784	0.1632	0.0724	0.9272	0.9314	0.9904
$\lambda_1 = 0.035$	0.1845	0.1629	0.1598	0.0510	0.9237	0.9277	0.9891
Experiments							
$\sigma = 0.0786$	0.2747	0.5806	0.0730	0.4313	0.8360	0.8466	0.9829
$\lambda_0 = 6084$							
$\lambda_1 = 0.056$	0.1810	0.1344	0.1408	0.0086	0.9051	0.9095	0.9859
$\sigma = 0.0786$							
$\lambda_1 = 6084$	0.2404	0.4866	0.0525	0.2277	0.7536	0.7663	0.9706
$\lambda_2 = 0.056$							

<sup>a</sup> Figures reported for  $K$ ,  $Y$  and  $TFP$  are ratios of aggregate capital, output, and output-weighted productivity to their costless counterparts

problem with costless external finance is also re-solved to compute the ratios of aggregate productivity, aggregate capital stock and aggregate output to their costless counterparts. Smaller ratios imply more severe adverse effects of costly external finance. Table 6 summarizes the results. The middle row of each panel corresponds to results from the baseline calibration.

Table 6 suggests that the adverse impacts of costly external finance on aggregate productivity and aggregate capital accumulation increase with the price of capital, the return to scale ( $\alpha$ ), the persistence ( $\rho$ ) and variability in

idiosyncratic productivity shocks ( $\sigma$ ) and external finance costs ( $\lambda_1$  and  $\lambda_2$ ). Some intuitions are as follows. First, an increase in the price of capital implies that financing investment in capital becomes more challenging for firms so that the adverse impact of costly external finance becomes even more severe. Second, an increase in  $\alpha$  implies a lower mark-up or lower operating profits for firms, so firms are more likely to resort to external finance for their investment needs. This is reflected in the higher external finance ratio corresponds to a higher  $\alpha$  in Table 6. Consequently costly external finance imposes more severe

adverse impacts on the economy. Similarly, an increase in  $\sigma$  implies more volatile productivity shocks, so firms are less able to hedge the higher uncertainty using internal funds and hence more heavily dependent on external finance. Finally, An increase in  $\rho$  leads to similar effects as an increase in  $\sigma$ , because a higher  $\rho$  implies higher volatility in firm level productivities (standard deviation of  $z$  is given by  $\frac{\sigma}{\sqrt{1-\rho^2}}$ ).

One implication of the comparative statics is that the result described in Sect. 4.1 may underestimate the quantitative impact of costly external finance on aggregate productivity growth for the US manufacturing, due to the nature of the Compustat data we use to calibrate the model. Since firms in our data sample are typically large and mature firms that are publicly traded, they are likely to exhibit less variability in productivities than a richer data set for US manufacturing like LRD would suggest and face lower external finance costs than an average manufacturing firm would face.

Another implication is that the adverse impact of costly external finance on productivity growth could be worse in recessions when uncertainty rises and external finance costs are also generally higher. As documented in Bloom et al. (2009), uncertainty at the establishment, firm, industry and aggregate level all rise significantly during recessions. In particular, their calibrated model suggests that in the recent economic downturn the volatility of firm-level TFP shocks has almost doubled. On the other hand, there has been a sharp rise in external finance premiums for both banks and nonbank borrowers in the recent credit crunch. Next, we perform a few experiments to illustrate how the rise in firm-level uncertainty and external finance costs in the recent credit crunch may exacerbate the quantitative impact of costly external finance. In the first experiment, we set  $\sigma$  to double its baseline value and keep all other parameters at baseline values. In the second experiment, we increase  $\lambda_0$  by 10-fold and double  $\lambda_1$  while keeping other parameters at their baseline values. The general magnitude of the rise in external finance premiums in the recent credit crunch is yet to be closely measured, though studies have documented tougher terms and higher interest rates for borrowing of many kinds (see Duchin et al. 2009 for example). The increases in  $\lambda_0$  and  $\lambda_1$  considered above are moderate considering the skyrocketing risk premiums in the Libor rates during the financial crisis. In the third experiment, we change  $\sigma$ ,  $\lambda_0$  and  $\lambda_1$  all together and keep other parameters unchanged. We again report the moments and ratios to costless counterparts from the experiments in Table 6. The results show that the rise in uncertainty alone leads to a 1.7 percent loss in aggregate productivity and the rise in external finance costs leads to a 1.4 percent loss. A joint rise in uncertainty and external finance costs,

however, causes a 3 percent loss in aggregate productivity, which triples the impact of costly external finance in the baseline case. These results suggest that the adverse impact of costly external finance could have exacerbated dramatically in the recent credit crunch, though our analysis is limited by the abstraction from aggregate uncertainty (which has gone up by fourfold according to Bloom et al. 2009).

## 5 Discussion

In this section, we discuss how our result concerning the quantitative impact of costly external finance on aggregate productivity may change if two important dimensions the model abstracts from, capital adjustment cost and firm entry and exit, are taken into account. The robustness to a general equilibrium analysis is also briefly discussed at the end.

### 5.1 Considering capital adjustment cost

Given the extensive literature on capital adjustment cost, it's worth to discuss the potential effect of adding it to the baseline model. Intuitively, with capital adjustment cost, investment in capital becomes more costly. On one hand, this calls for additional need of external funds to finance the adjustment cost so that costly external finance would impose a more severe impact on the economy. On the other hand, this could reduce investment in capital and hence reduce the demand of external finance, which would alleviate the adverse impact of costly external finance. The overall effect may go either way, depending on the nature and magnitude of capital adjustment cost. Next, we add a simple form of capital adjustment cost to the baseline model and examine its quantitative impacts.

Both convex and non-convex adjustment costs of capital are identified in various studies, see Cooper and Haltiwanger (2006) and Bloom (2009) for examples. A fixed adjustment cost, however, cannot be identified if it's included in the model, due to the presence of a fixed external finance cost. So we consider a standard convex adjustment cost, and the firm's problem is re-formulated as:

$$\begin{aligned}
 V(k, z) = \max_{k' \geq 0} & \quad \pi(k, z) - pi - \frac{\gamma}{2} \left(\frac{i}{k}\right)^2 k \\
 & - \lambda_0 I \left\{ pi + \frac{\gamma}{2} \left(\frac{i}{k}\right)^2 k > \pi(k, z) \right\} \\
 & - \lambda_1 \max \left\{ pi + \frac{\gamma}{2} \left(\frac{i}{k}\right)^2 k - \pi(k, z), 0 \right\} \\
 & + \beta E_{z'|z} V(k', z'), \tag{3}
 \end{aligned}$$



**Table 7** Aggregate productivity and moments with capital adjustment cost

Convex capital adjust cost		
$\gamma$	0.1647	0.2347
Re-calibration		
$\delta$	0.17	0.17
$\alpha$	0.8796	0.8667
$\rho$	0.9621	0.9588
$\sigma$	0.0646	0.0790
$\lambda_0$	5665.0	1601.9
Matched moments		
$I/K$	0.1704	0.1704
Avg. of $i/k$	0.1882	0.1890
Std. of $i/k$	0.2200	0.2219
Corr. of $i/k$	0.2161	0.2430
Extfin. $I$	0.0718	0.0720
Aggregate productivity	1.2982	1.3667
Ratio to costless counterpart	0.9919	0.9927

where  $i = k' - (1 - \delta)k$  and  $\gamma > 0$ . It's hard to identify  $\gamma$  in the presence of external finance costs using our investment series data, since both capital adjustment and external finance costs contribute to investment dynamics. Rather than resorting to additional data series or moments, we simply choose a value for  $\gamma$  and determine other parameters similarly as in the baseline calibration. We use the estimates of  $\gamma$  in Cooper and Ejarque (2003), in which several special versions of the model laid out in (3) are estimated using moments from Compustat data.<sup>14</sup> In one case, firms have free access to external finance, which corresponds to the costless external finance case. In another case, external finance is prohibitively expensive such that firms have no access to external finance at all. Their estimates of  $\gamma$  are 0.1647 and 0.2307, respectively. The model in (3) represents a case in-between: access to external finance is costly but not prohibitive. So we consider these two values for  $\gamma$ , and for each value of  $\gamma$  we re-estimate other parameters following the same procedure as described in Sect. 3, again using the simulated annealing algorithm. The new parameter estimates, matched moments, and productivity measures for the two values of  $\gamma$  are reported in Table 7.

Compared with the baseline calibration, a notable difference is that the estimates of  $\sigma$  are much larger, suggesting that with capital adjustment cost a larger variability of productivity shocks are needed to match the same investment dynamics. The matched moments for both values of  $\gamma$  are close to the data moments reported in Table 3. The loss in output-weighted aggregate productivity due to costly

<sup>14</sup> In Cooper and Ejarque (2003), the data moments also include autocorrelations and standard deviations of investment rates which take similar values as in our study.

external finance is 0.81 percent for the lower  $\gamma$  case and 0.73 percent for the higher  $\gamma$  case. These results are slightly lower than the productivity loss implied by the baseline model, but are still quantitatively comparable.

## 5.2 Considering firm entry and exit

The Compustat data set used to estimate the model does not exhibit a lot of firm entry and exit, but entry and exit is a common behavior of the US manufacturing industry. According to Dunne et al. (1988), on average approximately 4.5 percent of firms entered the US manufacturing industry every year during the period of 1963–1982 and a similar percentage of firms exited every year. Empirical studies also find a significant role of entry and exit of production units in accounting for aggregate productivity growth. This subsection presents a brief discussion of how the quantitative impact of costly external finance on aggregate productivity may change if adding firm entry and exit to the model. Rather than doing a comprehensive analysis, we consider some simple cases of firm entry and exit.

Assume that the firm's exit is exogenous: every period, the firm has a probability of  $\eta$  to exit, where  $\eta = 0.045$ .<sup>15</sup> Upon exit, the firm secures a zero exit value. Now the firm's problem is determined by

$$\begin{aligned}
 V(k, z) = \max_{k' \geq 0} & \quad \pi(k, z) - pi(k, k') \\
 & \quad - \lambda_0 I\{pi(k, k') > \pi(k, z)\} \\
 & \quad - \lambda_1 \max\{pi(k, k') - \pi(k, z), 0\} \\
 & \quad + \beta(1 - \eta)E_{z'|z}V(k', z').
 \end{aligned} \tag{4}$$

In the data, there are high-productivity entrants and low-productivity entrants. So we consider two extreme cases of firm entry to infer the impact of entry and exit. First, as in Cooley and Quadrini (2001), new entry firms are of the highest productivity, and second, new entry firms are of the lowest productivity.<sup>16</sup> Upon entry, a new firm chooses its initial capital stock, which is financed all by external funds, to maximize its expected continuation value. The entry problem is formulated as:

<sup>15</sup> There is evidence that firm exits are related to low productivity, and also impacted by external financing issues. Some recent literature on firm dynamics has explicitly modeled these links, see Hopenhayn (1992) and Clementi and Hopenhayn (2006) for examples. Modeling these issues is beyond the scope of the paper.

<sup>16</sup> A more realistic way to model firm entry is to let new firms' productivity follow some distribution. We abstract from this complication since results from the two simple extreme cases would somehow provide a range for the quantitative impact of costly external finance in an environment where firm exit is exogenous and new entry firms' productivities range from the lowest to the highest level, and in our view this is sufficient to provide some insights on the sensitivity of the results to firm entry and exit.

$$V_0(z_0; p) \equiv \max_{k_0} \int V(k_0, z') P(z_0, dz') - \lambda_0 - p(1 + \lambda_1)k_0, \tag{5}$$

where  $z_0 = \bar{z}(z)$  for the first (second) case. Free entry condition implies that

$$V_0(z_0; p) = c_e, \tag{6}$$

where  $c_e$  is a fixed entry cost.

Then for each case, we estimate the model with entry and exit following the procedure described in Section 3. In each case,  $c_e$  is chosen such that the free entry condition (6) is satisfied. To solve the corresponding costless problem, we let  $\lambda_0 = \lambda_1 = 0$  in problem (4) and (5), and choose the price of capital good,  $p^c$ , such that (6) is satisfied. The results for the two cases are reported in the first half of Table 8. Note that the matched moments in both cases are reasonably close to the data moments in Table 3, except the standard deviation of investment rates in Case 1.<sup>17</sup> The results show that the ratio of output-weighted aggregate productivity to its costless counterpart is 0.9982 if new firms are of the highest productivity, and 0.9922 if new firms are of the lowest productivity. In both cases, the loss in output-weighted aggregate productivity due to costly external finance is less than 1 percent.

One may argue that the re-calibration here is not very appropriate since a model that considers firm entry and exit should be calibrated to a richer data set that exhibits a lot of firm entry and exit. For a robustness check, we also consider a crude calibration of the model to LRD. Of course, due to the lack of access to LRD and the lack of financial data in LRD, we are not able to compute the exact moments for firms covered in the LRD that are needed for calibration. So we use the moments of plant level investment rates reported in Cooper and Haltiwanger (2006) (LRD, 1972–1988, mean: 12.2%, standard deviation: 33.7%, autocorrelation: 5.8%) as proxies for moments of firm level investment rates, and the fraction of external funds in the sources of funds reported in Fazzari et al. (1988) (US manufacturing firms, 1970–1984, 28.9%) as the external finance ratio of LRD firms. Note that values of these moments are quite different from what we use in previous calibrations. In particular, standard deviation of investment rates and external finance ratio are much higher, which is probably true for LRD firms as compared to the Compustat firms. We let  $\delta = 0.1, \beta$  and  $\lambda_1$  be the same as before, and estimate  $\alpha, \rho, \sigma$  and  $\lambda_0$  to match the four moments. The results under the new calibration are

<sup>17</sup> The estimation routine finds that there is a tension in the two moments: standard deviation of investment rates and external finance ratio. Since we put more emphasis on external finance ratio, as we do for the baseline model, the estimation yields a low standard deviation of investment rates than in the data.

**Table 8** Aggregate productivity and moments with firm entry and exit

	Case 1: new firms have the highest TFP	Case 2: new firms have the lowest TFP	
<b>Re-calibration to the Compustat data</b>			
$\delta$	0.17	0.17	
$\alpha$	0.9451	0.9028	
$\rho$	0.5922	0.8958	
$\sigma$	0.013	0.0324	
$\lambda_0$	4551.7	1399.4	
<b>Matched moments</b>			
$I/K$	0.1698	0.1703	
Avg. of $ilk$	0.1972	0.2247	
Std. of $ilk$	0.0909	0.1732	
Corr. of $ilk$	0.1253	0.1912	
Extfin. $I$	0.1299	0.0718	
Aggregate productivity	1.0031	1.0096	
Ratio to costless counterpart	0.9982	0.9922	
<b>A crude calibration to LRD</b>			
$\delta$	0.1	0.1	
$\alpha$	0.7576	0.8762	
$\rho$	0.7502	0.9151	
$\sigma$	0.0963	0.0486	
$\lambda_0$	2129.2	1237.1	
<b>Matched moments</b>			
$I/K$	0.0998	0.1004	LRD
Avg. of $ilk$	0.1114	0.1678	0.122
Std. of $ilk$	0.1878	0.3036	0.337
Corr. of $ilk$	0.0588	0.0791	0.058
Extfin. $I$	0.2684	0.2833	0.289
Aggregate productivity	1.0992	1.0396	
Ratio to costless counterpart	0.9902	0.9860	

reported in the bottom half of Table 8. Note that the ratio of aggregate productivity to its costless counterpart is 0.9902 if new firms are of the highest productivity, and 0.9860 if new firms are of the lowest productivity. As expected, the loss in aggregate productivity due to costly external finance is larger than obtained from the models calibrated to the Compustat data. Nevertheless, it's clear that the quantitative magnitude does not deviate from the baseline result substantially.

### 5.3 Considering general equilibrium

In our partial equilibrium analysis, firms take a constant price of capital as given. In the calibration, we normalize this price to be 1 while choosing other parameters to match certain data moments. Table 6 shows that with other



parameters unchanged, the percentage loss in aggregate productivity due to costly external finance increases with the price of capital. However, once we re-calibrate other parameters to match the data moments, a similar result as in the baseline case would be obtained. For example, letting  $p = 1.2$  and re-calibrating other parameters yield a ratio of output-weighted aggregate productivity to its costless counterpart equal to 0.9905. This result suggests that our main quantitative result should be robust to a general equilibrium analysis without aggregate uncertainty, in which the price of capital would be endogenized, but remain as a constant.

A general equilibrium analysis, however, would allow us to address some other interesting issues, such as the quantitative impact of costly external finance on aggregate output and welfare. Further, by focusing on a stationary analysis, we ignore the possible fluctuations in external finance costs and price of capital over business cycle. There is empirical evidence that external finance premium is countercyclical. Such cyclical fluctuations could have important implications for productivity growth, both at firm and aggregate level, though a model with heterogeneous firms and aggregate shocks would be much more complex and computationally challenging.

## 6 Final remarks

This paper studied the quantitative impact of costly external finance on aggregate productivity by incorporating an external finance cost function into the dynamic optimization problem of firms with idiosyncratic productivity shocks. Our main result was that costly external finance leads to a reallocation of output shares from high productivity firms to low productivity firms such that the output-weighted aggregate productivity is 1 percent smaller than it would be if external finance is not costly. By lowering aggregate productivity, costly external finance decreases aggregate output by 0.3 percent. This is an indirect impact of costly external finance on aggregate output, in addition to its direct impact through reducing aggregate capital accumulation.

We abstracted from firm entry and exit in the main analysis and considered exogenous entry and exit in the discussion. As entry and exit plays an important role in productivity growth and finance matters for entry and exit, an interesting extension of the paper is to model how costly external finance affects firms' entry and exit decisions and quantitatively evaluate this impact on aggregate productivity. In addition, we adopted a homogeneous external finance cost function. There is empirical evidence suggesting that firms differ in external finance costs along a lot of dimensions, such as firm size, age, and credit

worthiness. Further, we excluded demand and cost shocks that may also affect firms' profitability and investment dynamics. This could call into question the identification of productivity shocks by moments describing investment dynamics. A richer model that allows for other important profitability shocks and a richer data set are needed to provide a better identification of productivity dynamics. These issues are open for future research.

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